\*\*1. Probability of first upgrade after the third flight:\*\*

Let X be the random variable representing the number of flights until Sam's first upgrade. This is a geometric distribution with probability of success (upgrade) p = 0.10. We want to find P(X > 3), which is the probability that the first upgrade occurs after the third flight. This is equivalent to the probability of no upgrades in the first three flights. Since the flights are independent, we have:

P(X > 3) = (1 - p)³ = (1 - 0.10)³ = (0.9)³ = 0.729

Therefore, the probability that Sam's first upgrade will occur after the third flight is 0.729.

\*\*2. Probability of exactly 2 upgrades in 20 flights:\*\*

The number of upgrades in 20 flights follows a binomial distribution with n = 20 trials and probability of success p = 0.10. We want to find P(Y = 2), where Y is the number of upgrades in 20 flights. The probability mass function for a binomial distribution is:

P(Y = k) = (n choose k) \* p^k \* (1 - p)^(n-k)

So, for k = 2:

P(Y = 2) = (20 choose 2) \* (0.10)² \* (0.90)^(20-2) = 190 \* (0.01) \* (0.90)^18 ≈ 0.285

Therefore, the probability that Sam will be upgraded exactly 2 times in his next 20 flights is approximately 0.285.

\*\*3. Surprise at more than 20 upgrades in 104 flights:\*\*

Let Z be the number of upgrades in 104 flights. This also follows a binomial distribution with n = 104 and p = 0.10. The expected number of upgrades is E(Z) = np = 104 \* 0.10 = 10.4. The variance is Var(Z) = np(1-p) = 104 \* 0.10 \* 0.90 = 9.36. The standard deviation is sqrt(9.36) ≈ 3.06.

We want to assess the probability of Z > 20. We can approximate this using a normal distribution since n is large:

Z ≈ N(10.4, 9.36)

We can standardize to find the z-score:

z = (20 - 10.4) / 3.06 ≈ 3.14

Using a standard normal table or calculator, P(Z > 3.14) is very small (approximately 0.0008).

\*\*Conclusion:\*\*

It would be highly surprising if Sam received more than 20 upgrades in 104 flights. The probability is extremely low, based on the normal approximation to the binomial distribution. The observed outcome would be highly unlikely under the assumption that the airline's claim of a 0.10 upgrade probability is accurate. This would warrant further investigation into the airline's claim or Sam's flying patterns (e.g., perhaps his flight choices systematically bias him towards flights with higher upgrade rates).